Absolute Value Acrobatics

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Definition 1. “the absolute value of x” is how the following expression is read: “$|x|$”.

Definition 2. $|x| =$ x if $0 \leq x$
$-x$ if $x < 0$.

Synapse 1. $|x| = 0 \leftrightarrow x = 0$.

Synapse 2. $|-x| = |x|$.

Synapse 3. $|xy| = |x||y|$.

Synapse 4. $-|x| \leq x \leq |x|$.

Synapse 5.1. $|x| \leq a \leftrightarrow -a \leq x \leq a$.

Synapse 5.2. $|x| < a \leftrightarrow -a < x < a$.

Synapse 6. $a > 0 \rightarrow (|x| = a \leftrightarrow (x = a \vee x = -a))$

Synapse 7. $\max\{x,y\} = (1/2)(x + y + |x - y|)$

Synapse 8. $\min\{x,y\} = (1/2)(x + y - |x - y|)$

Theorem 1. $|x + y| \leq |x| + |y|$. (subadditivity)

Theorem 2. $|x - y| \leq |x| + |y|$. (triangle inequality)

Theorem 3. $|x| - |y| \leq |x - y|$.

Theorem 4. $||x| - |y|| \leq |x - y|$.

Regarding equations involving absolute value, note that $|x| = |y|$ if, and only if, $x = y$ or $x = -y$, but $|x| = y$ implies $x = y$ or $x = -y$, that is, the latter is a one-way implication, and therefore using it requires checking for extraneous roots.

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