Arithmetic Progressions

Definition 1. **Three given numbers** are said to be in **arithmetic progression** if, and only if, the middle number is the arithmetic mean of the other two numbers.

Definition 2. A **baby arithmetic progression** (abbreviated "bap") is a 3-tuple (x,y,z) of numbers such that $x \leq y \leq z$ or $z \leq y \leq z$ and the three numbers are in arithmetic progression.

Theorem 1. If each of x, y and z is a number, then (x,y,z) is a baby arithmetic progression if, and only if, (z,y,z) is a baby arithmetic progression.

Theorem 2. If each of x, y and z is a number, then (x,y,z) is a baby arithmetic progression if, and only if, y is the arithmetic mean of x and z.

Theorem 3. If each of x, y and z is a number, then (x,y,z) is a baby arithmetic progression if, and only if, the difference between consecutive terms is constant, that is, $y - x = z - y$.

Definition 3. A **progression** is any **assignment scheme** for the positive integers. (It is customary to subscript the positive integers to the name of the assignment scheme to indicate the terms of the progression, for example, if "T" is the name of the assignment scheme, then the terms of the progression are T_1, T_2, T_3, and so on.)

Definition 4. A progression of numbers is said to be **arithmetic** if, and only if, every three consecutive terms constitutes a baby arithmetic progression.

Theorem 4. A progression of numbers is arithmetic if, and only if, the difference between any two consecutive terms is constant, that is, if, and only if, there exists a number d such that the difference between any two consecutive terms is d.

Theorem 5. A progression T_1, T_2, T_3, ... of numbers is arithmetic if, and only if, there exists a number **a** and a number **d** such that for each positive integer n, $T_n = a + (n - 1)d$.

Theorem 6. The **sum** of the first n terms of an arithmetic progression is equal to n times the arithmetic mean of the **first** and **last** terms of the summation, that is, is equal to $n((F + L)/2)$, where n is a positive integer, and where $F = a$ and $L = a + (n - 1)d$, where a and d are numbers such that for each positive integer k, the k-th term of the progression is equal to $a + (k - 1)d$.

Theorem 7. If T_1, T_2, T_3, ... is a progression of numbers such that the sum of the first n terms of the progression is equal to n times the arithmetic mean of the first and last terms of the summation, for each positive integer n, then T_1, T_2, T_3, ... is an arithmetic progression.

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