

Average Annual Growth Rate Explanation

Let P be the amount of dollars put into an account. Let r_1 be the rate by which the fund has grown after one year, and let r_2 be the additional rate by which the fund has grown after yet another year. Let Q be the amount of dollars in the account after these two years. Then $Q = P(1 + r_1)(1 + r_2)$.

Definition 1. The average annual growth rate, for this two-year period, is that rate r such that $Q = P(1 + r)(1 + r)$.

Theorem 1. $r = \sqrt{(1 + r_1)(1 + r_2)} - 1$.

Proof: Left to the reader. ■

Theorem 2. $r = \sqrt{Q/P} - 1$.

Proof: Because $Q/P = (1 + r_1)(1 + r_2)$. ■

Which theorem (formula) you use depends on whether you are given r_1 and r_2 , or are given P and Q .

Notice that we do not simply take the average (that is, the arithmetic mean) of r_1 and r_2 . Indeed, we have the following theorem.

Theorem 3. If $r_1 \neq r_2$, then $r < (r_1 + r_2)/2$.

Proof: Left to the reader. ■

Note: The average annual growth rate is also known as the **Compound Annual Growth Rate (CAGR)**. The latter expression has the advantage of not suggesting the use of the arithmetic mean.

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