

The Birthday Problem

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This is a topic in Probability Theory.

Problem: What is the probability that at least 2 people out of a given group of 23 randomly chosen people have the same birthday?

Solution: Let A be the event that at least 2 people have the same birthday. Let B be the complement of A. Then B is the event that no 2 people have the same birthday. The probability that 2 people picked from the 23 do not have the same birthday is $364/365$. (To see this, suppose that 1 person is first picked from the 23, and then a second person is picked. The probability that the 2 people do not have the same birthday is the same as the probability that the birthday of the second person is different from that of the first person, and that probability is obviously $364/365$.) So, the probability of B is $(364/365)^{23}$ raised to the power of the number of ways that this happens, which is $C(23,2)$, that is, the number of combinations of 23 things taken 2 at a time, which equals $23!/(2! \times (23 - 2)!)$, which equals 253. Thus, the probability of B is $253 \times (364/365)^{23}$. Therefore, the probability of A is $1 - 253 \times (364/365)^{23}$, which equals 0.500047. Thus, for merely 23 people, there is already more than a 50% probability that 2 of them have the same birthday.

For more information on this famous problem, see the Wikipedia article on “Birthday Problem”.

Note: $b@a$ means a to the power b. For example, $3@10 = 1000$. This notation allows you to avoid using superscripts.

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