

Canonical Algebraic Exercise

Notation: “ \vee ” means “or”.

Notation: “ \leftrightarrow ” means “if, and only if,”.

Zero Principle: $ab = 0 \leftrightarrow (a = 0 \vee b = 0)$.

Definition of Subtraction: $x - y = x + (-y)$.

Lemma: $(x + 2)(x - 3) = x^2 - x - 6$.

Exercise: Solve for x:

$$\sqrt{|x^2 - x - 6|} = 0$$

Solution:

$$\sqrt{|x^2 - x - 6|} = 0$$

$$\leftrightarrow |x^2 - x - 6| = 0 \quad ; \quad a = 0 \leftrightarrow a^2 = 0$$

$$\leftrightarrow x^2 - x - 6 = 0 \quad ; \quad |a| = 0 \leftrightarrow a = 0$$

$$\leftrightarrow (x + 2)(x - 3) = 0 \quad ; \quad \text{Lemma}$$

$$\leftrightarrow x + 2 = 0 \vee x - 3 = 0 \quad ; \quad \text{Zero Principle}$$

$$\leftrightarrow x = -2 \vee x = 3 \quad ; \quad \text{manipulation}$$

$$\leftrightarrow x \in \{-2, 3\} \quad ; \quad \text{set-theoretic notation}$$

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