

The Derivative of $\frac{i^3}{3}$

$$\left(\frac{i^3}{3}\right)' \cdot x = \lim_{h \rightarrow 0} \frac{\left(\frac{i^3}{3}\right) \cdot (x+h) - \left(\frac{i^3}{3}\right) \cdot x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\left(\frac{(x+h)^3}{3} - \frac{x^3}{3}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{3} \left((x+h)^3 - x^3 \right)}{h}$$

$$= \frac{1}{3} \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{1}{3} \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$$

$$= \frac{1}{3} \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \frac{1}{3} \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2)$$

$$= \frac{1}{3} \left(\lim_{h \rightarrow 0} 3x^2 + \lim_{h \rightarrow 0} 3xh + \lim_{h \rightarrow 0} h^2 \right)$$

$$= \frac{1}{3} (3x^2 + 0 + 0)$$

$$= \left(\frac{1}{3}\right) (3x^2)$$

$$= x^2.$$

$$\therefore \left(\frac{i^3}{3}\right)' = i^2.$$