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DC (Direct Current) Circuit

Consider an electrical circuit containing a battery, a resistor, and an inductor in series. Suppose that the battery generates a potential difference of V, and the resistor has a resistance of R, and the inductor has an inductance of L. It is essential to be aware that an inductor poses a resistance to the current proportional to the rate of change of the current. (L is the constant of proportionality.) We will use "I" to stand for the function giving the amount of current flowing in the circuit at a given time. That is, I.t is the current in the circuit at time t. We take I.0 to be 0. Our goal: Obtain a formula for I in terms of known quantities. Note that we consider V, R, and L to be known. However, we also consider other things to be known, such as the exponential function (ε) and the identity function (i), which we can use freely along with V, R, and L in stating the final formula. We now begin the analysis leading to our goal. The voltage drop across the resistor is RI, and the voltage drop across the inductor is LI'. Their sum must equal the total voltage drop V. Therefore, $$LI' + RI = V.$$ Therefore, dividing through by L, we obtain $$I' + \frac{R}{L} I = \frac{V}{L}.$$	
But this is a first order linear differential equation: $$f' + Pf = Q,$$	750800
in which f = I, P = R/L, Q = V/L, and f.0 = 0.	
Therefore, the solution is $$I = \frac{V}{R}\,(1 - \varepsilon..(\frac{-Ri}{L})).$$ The above formula constitutes reaching our goal.	