Flora's Theorem 1 on Arithmetic Progressions

Theorem 1. For an arithmetic progression whose general term T_n is $a + (n - 1)d$,

$$a = \frac{S_2 - d}{2}.$$

Proof: $S_2 \equiv T_1 + T_2 = T_1 + (T_1 + d) = 2T_1 + d = 2a + d$.

Therefore, $2a + d = S_2$.

Therefore, $a = \frac{S_2 - d}{2}$. $\blacksquare$

Example. Find an arithmetic progression such that $S_2 = 5$.

$d = -3 \rightarrow a = \frac{5 - -3}{2} = 4$: 4, 1, -2, -5, …

$d = -2 \rightarrow a = \frac{5 - -2}{2} = \frac{7}{2}$: 7/2, 3/2, -1/2, -5/2, …

$d = -1 \rightarrow a = \frac{5 - -1}{2} = 3$: 3, 2, 1, 0, …

$d = 0 \rightarrow a = \frac{5 - 0}{2} = \frac{5}{2}$: 5/2, 5/2, 5/2, 5/2, …

$d = 1 \rightarrow a = \frac{5 - 1}{2} = 2$: 2, 3, 4, 5, …

$d = 2 \rightarrow a = \frac{5 - 2}{2} = \frac{3}{2}$: 3/2, 7/2, 11/2, 15/2, …

$d = 3 \rightarrow a = \frac{5 - 3}{2} = 1$: 1, 4, 7, 10, …

$d = 4 \rightarrow a = \frac{5 - 4}{2} = \frac{1}{2}$: 1/2, 9/2, 17/2, 25/2, …

$d = 5 \rightarrow a = \frac{5 - 5}{2} = 0$: 0, 5, 10, 15, …

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