# Flora's Theorem 2 on Arithmetic Progressions

Theorem 2. For an arithmetic progression whose general term $T_n$ is $a + (n - 1)d$,

$$a = \frac{3S_2 - S_3}{3} \quad \text{and } d = \frac{2S_3 - 3S_2}{3}.$$

Proof: $S_2 \equiv T_1 + T_2 = T_1 + (T_1 + d) = 2T_1 + d = 2a + d$,
and
$S_3 \equiv T_1 + (T_1 + d) + ((T_1 + d) + d) = 3T_1 + 3d = 3a + 3d$.
Thus, $2a + d = S_2$ and $3a + 3d = S_3$.
Solving these two equations simultaneously yields the stated result. ∎

Example. Find the arithmetic progression such that $S_2 = 5$ and $S_3 = 10$.

$$a = \frac{3 \times 5 - 10}{3} = \frac{15 - 10}{3} = \frac{5}{3}$$

and

$$d = \frac{2 \times 10 - 3 \times 5}{3} = \frac{20 - 15}{3} = \frac{5}{3}.$$

The progression is therefore: $\frac{5}{3}, \frac{10}{3}, \frac{15}{3}, \frac{20}{3}, \ldots$

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