

## The GM-AM Inequality

**Theorem:** If  $x_1, \dots, x_n$  are positive quantities, then

$$\left( \prod_{k=1}^n x_k \right)^{\frac{1}{n}} \leq \frac{1}{n} \sum_{k=1}^n x_k .$$

**Proof:** The proof is difficult, and is not included here. [See, for example, [Introduction to Inequalities](#) by Beckenbach and Bellman.]

The quantity on the left in the above inequality is called the Geometric Mean (GM), and the quantity on the right is called the Arithmetic Mean (AM).

**Example.** Let  $n = 3$ ,  $x_1 = 5$ ,  $x_2 = 8$ ,  $x_3 = 21$ . Then  $GM = 6.80$ , and  $AM = 9.67$ , and we note that it is a true statement that  $GM \leq AM$ .

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