The Napierian Calculus Component

Substitution Theorem for Integrals:

$$\int_a^b (f \circ g)\, g' = \int_{g.a}^{g.b} f \,.$$

Definition of the Identity Function: $\forall x,\ i.x = x$.

Integral Invariance Theorem:

$$\int_x^{xy} \frac{1}{i} = \int_1^y \frac{1}{i} \,.$$

Proof:

$$\int_x^{xy} \frac{1}{i} = \int_{(xi).1}^{(xi).y} \frac{1}{i} = \int_1^y \left(\frac{1}{i} \circ (xi)\right) (xi)' = \int_1^y \left(\frac{1}{xi}\right)(x) = \int_1^y \frac{1}{i} \quad \blacksquare$$

Note 1: The Substitution Theorem for Integrals is also known as The Change-of-Variables Formula.

Note 2: The Integral Invariance Theorem is used in establishing the integral representation of the logarithm.