

## Summation Example 8271

Task: Find positive integers  $a$  and  $b$ , relatively prime to each other, such that  $\frac{a}{b} = 0.5\dot{8}$ .

Do this by explicitly using an infinite series that links to a geometric progression.

Solution: Let  $x = 0.5\dot{8}$ . Then  $x = \frac{5}{10} + 0.0\dot{8} = \frac{1}{2} + 8(0.0\dot{i})$

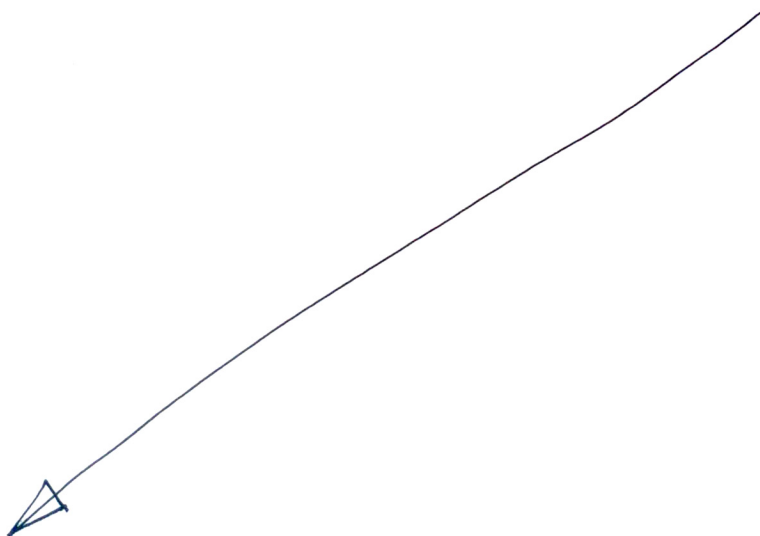
$$= \frac{1}{2} + \frac{8}{10} (0.\dot{i}) = \frac{1}{2} + \frac{4}{5} (0.\dot{i})$$

$$= \frac{1}{2} + \frac{4}{5} \sum_{k=1}^{\infty} \left(\frac{1}{10}\right)^k = \frac{1}{2} + \frac{4}{5} \sum_{k=1}^{\infty} \left(\frac{1}{10}\right) \left(\frac{1}{10}\right)^{k-1}$$

$$= \frac{1}{2} + \frac{4}{5} \text{LS} \left( \text{GP} \left( \frac{1}{10}, \frac{1}{10} \right) \right)$$

$$= \frac{1}{2} + \frac{4}{5} \left( \frac{\frac{1}{10}}{1 - \frac{1}{10}} \right) = \frac{1}{2} + \frac{4}{5} \left( \frac{1}{10} \times \frac{10}{9} \right)$$

$$= \frac{1}{2} + \frac{4}{5} \times \frac{1}{9} = \frac{1}{2} + \frac{4}{45} = \frac{45}{90} + \frac{8}{90} = \frac{53}{90}.$$



Thus,  $a = 53$  and  $b = 90$ .

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