

## The product of two binomials

A binomial is an algebraic expression containing exactly 2 terms.

$$(x + a)(y + b)$$

$$= xy + ay + bx + ab$$

if  $y = x$ , then

$$= x^2 + (a + b)x + ab$$

if  $y = x$  and  $b = a$ , then

$$= x^2 + 2ax + a^2 \quad ; \text{mountain}$$

if  $y = x$  and  $b = -a$ , then

$$= x^2 - a^2 \quad ; \text{river}$$

**mountain:** The third term is the square of half the coefficient appearing as part of the middle term. This fact is used when one wants to "complete the square" in the course of algebraic manipulations.

**river:** The factorization  $x^2 - a^2 = (x - a)(x + a)$  specifies a rectangle having the same area as the difference of two given squares, namely, the rectangle has length equal to the sum of the lengths of the squares and width equal to the the difference of the lengths of the squares. (The related question of how to construct a rectangle - indeed, another square - having the same area as the **sum** two squares is solved by the Pythagorean Theorem. That is, put the two lengths of the squares perpendicular to each other. The distance between their far endpoints is the length of a square equal in area to the sum of the areas of the two given squares.) This also gives a nice pair of tongs for handling quadratic equations. However, since "x" and "a" typically appear at the detail level, we will use "P" and "Q" for the strategy. That is, we have the following factorization:  $P^2 = Q^2$  fif  $P = \pm Q$  which is used in along with the Zero Principle. (The Zero Principle states that the product of two number is 0 fif at least one of the numbers is 0. Putting this algebraically, we have:  $PQ = 0$  fif  $P = 0$  or  $Q = 0$ .) For example, suppose that we are given that  $x^2 - 5x + 6 = 0$ , and are asked to "solve for x" (or, "find x"). We use the fact that  $x^2 - 5x + 6 = (x - 2)(x - 3)$ . Here,  $x - 2$  is P, and  $x - 3$  is Q. So, we have  $PQ = 0$ . So,  $P = 0$  or  $Q = 0$ . So,  $x - 2 = 0$  or  $x - 3 = 0$ . So,  $x = 2$  or  $x = 3$ . Done.

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