Trig Example 819025

Given that $f(x) \equiv 4\sin x + 2$,

then:

$f(2x) = 0 \wedge x \in [0, 360)$

$\Leftrightarrow \sin 2x = -\frac{1}{2} \wedge x \in [0, 360)$

$\Leftrightarrow 2x \in \{210 + 360k \mid k \in \mathbb{Z}\} \cup \{330 + 360k \mid k \in \mathbb{Z}\} \wedge x \in [0, 360)$

$\Leftrightarrow x \in \{105 + 180k \mid k \in \mathbb{Z}\} \cup \{165 + 180k \mid k \in \mathbb{Z}\} \wedge x \in [0, 360)$

$\Leftrightarrow x \in (\{105 + 180k \mid k \in \mathbb{Z}\} \cup \{165 + 180k \mid k \in \mathbb{Z}\}) \cap [0, 360)$

$\Leftrightarrow x \in (\{105 + 180k \mid k \in \mathbb{Z}\} \cap [0, 360)) \cup (\{165 + 180k \mid k \in \mathbb{Z}\} \cap [0, 360))$

$\Leftrightarrow x \in \{105, 285\} \cup \{165, 345\}$

$\Leftrightarrow x \in \{105, 165, 285, 345\}$

Notice that we use the fact that (set-theoretic) intersection distributes over (set-theoretic) union. That is,

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C).$$

(end)