Trigonometry Overview
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Many quantities depend on angle, and hence, consideration of angle is often central to our computational concerns. The scope of angle dependence might surprise you. For example, the probability that your pen, if tossed onto a tiled floor, will come to rest in a way that crosses one of the lines made by the tiles depends on the angle that your pen comes to rest at. (And since anything involving angles involves π, such repeated tosses constitutes a way of computing π.)

Trigonometry consists of the exploitation of the similarity of triangles in order to find out certain inaccessible quantities that depend on angle, or to express certain accessible quantities dependent on angle in an alternative or more convenient form.

Let's devise an artificial scenario to show, in a condensed time frame, how the subject of trigonometry could have developed. Let us suppose that you have been commissioned to find out the heights of trees. You are new at this, and have only Euclidean geometry and elementary algebra at your command. So, for your first tree, you position yourself at some convenient distance from the tree. (Let's call this distance "d".) You call the top of the tree "A", and you call your position "B", and you call the base of the tree "C". Triangle ABC is then a right triangle, and you wish to know the length of side CA, which we will call "h" (the height of the tree). (The length of side CB is d, which is a value known to you, indeed, chosen by you.) You measure angle ABC, which we will call "the angle of elevation", and denote by "θ", and then construct a small right triangle PBQ similar to the big right triangle ABC, such that P corresponds to A, and Q corresponds to C. Let us call the length of QB "x" and the length of PQ "y". Then by similarity of the two triangles, $h/d = y/x$. Therefore $h = d(y/x)$. Since d, x, and y are all known to you, you can now compute h. However, this was rather time-consuming, as you spent most of the time in the construction of the small triangle. Therefore, knowing that you have a multitude of similar (no pun intended) tasks ahead of you, you decide to create a lot of small triangles in advance. As you begin this background task for your tree-height measuring work, you make a crucial observation, namely, that you are needlessly expending energy by storing the information for all pieces of the small triangle. That is to say, you realize that you do not need to know the individual values of x and y, but only their quotient, and that this quotient depends only on the angle of elevation. You decide to tabulate, for various values of the angle of elevation, this quotient, which, to anticipate later terminology, we will call the tangent of the angle of elevation, that is the tangent of θ, which we will denote by "$\tan(\theta)$". Thus, your computation of the height of that first tree can be re-written as $h = d*\tan(\theta)$. You decide to compute the tangent of the angle of elevation for values of the angle of elevation from 0.01° to 89.99°. You put this information into a book that you carry with you. Now, you are able to quickly find out the height of a tree. You pick your distance d from the tree, measure the angle of elevation θ, and then look up the value of the angle in your book that is closest to θ and copy off the value of its tangent onto your scratch paper and do the computation $d*\tan(\theta)$, and you are done with that tree. Very slick.

You have just invented trigonometry, or at least established a beachhead. Much later, when you have finally invented calculus, you make the major discovery that it is not even necessary to construct the small triangles physically in order to get the values of the tangent and the other trigonometric functions, but rather, that these values can be computed from certain infinite series that calculus makes available to you.

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