

Two Theorems on Radian Angle Measure

Theorem 1. For a circle of radius r , the length of the arc subtended by an angle θ , measured in radians, is $r\theta$.

Proof: Consider a unit circle concentric with the given circle of radius r . Then, by similarity of figures, $\theta/1 = L/r$, where L is the length of the arc subtended by the angle θ for the circle of radius r . Then $L = r\theta$. ■

Theorem 2. For a circle of radius r , the area of the sector of a circle determined by an angle θ , measured in radians, is $(1/2)r^2\theta$.

Proof: Let w be the area of the sector. Then the ratio of w to the whole area of the circle equals the ratio of L (the length of the arc that θ is standing on) to the whole circumference of the circle. That is, $w/(\pi r^2) = L/(2\pi r)$. But by Theorem 1, $L = r\theta$. Thus, $w/(\pi r^2) = r\theta/(2\pi r)$. Thus, $w = (1/2)r^2\theta$. ■

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